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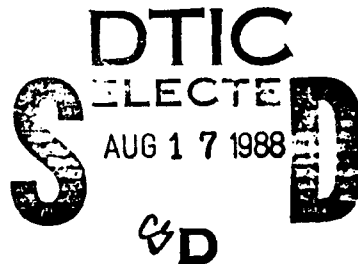
*Allocation*  
INTERCEPTOR-TO-TARGET STRATEGIES  
FOR STRATEGIC DEFENSE I:  
ADAPTIVE STRATEGIES BASED UPON SYSTEM  
AND THREAT CHARACTERISTICS

Maile E. Smith  
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## EXECUTIVE SUMMARY

This paper examines the topic of interceptor-to-target assignment algorithms and compares the system performance of a strategic defense system (SDS) as a function of the algorithm used. The algorithms used in this study range from being similar to some of those being developed under the SDIO's advanced algorithm programs to those used in some of the more popular engagement models. It is shown that the use of less sophisticated algorithms can often lead to RV leakages twice as large compared to cases in which sophisticated algorithms are used.

The primary conclusion from this paper is that as the defense becomes interceptor-rich and/or the Pk of the interceptor becomes low, the popular "one-interceptor-per-target" approaches to interceptor-allocation, which are typically found in the community-wide engagement models, can perform poorly. When the defense is in either, or both, of these regimes, it can perform significantly better by adopting an approach in which the salvo becomes a credible strategy. Because the salvo strategies have not been implemented in the popular engagement models, the past analyses which have studied these types of architectures have been biased toward providing a lower level of performance.

It is shown that the algorithms for boost/post-boost interceptor allocation embedded in the popular engagement models, including those used in the System Analyses and Key Trade-off Studies, are most appropriate to the study of interceptor-poor architectures, and thus are appropriate for analyses on the Phase 1 SDS. Conversely, it is shown that these same algorithms lead to very pessimistic predictions about leakage through the post-boost phase, compared to more robust algorithms, for an interceptor-rich defense. It is suggested that some of the previous studies on the interceptor-rich architectures be revisited with better algorithms.

Interceptor allocation strategies for the midcourse and terminal phase are not analyzed in this paper.

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## I. INTRODUCTION

The complete definition of any Strategic Defense System requires more than a listing of hardware components and the performance requirements which they must meet. Equally important, but yet unspecified aspects of the architecture definition are the algorithms governing complex processes such as track formation and maintenance, discrimination, and interceptor allocation. Because no battle management algorithms have been fully designed and accepted, all previous studies of architecture performance necessarily relied on certain assumptions regarding how well and how quickly the defense might perform tracking and discrimination. In the case of interceptor assignment, however, most studies made use of an actual interceptor-assignment algorithm which was embedded in the available engagement model. Needless to say, there is no universal agreement on the validity of the assumptions regarding the tracking and discrimination processes, nor on what interceptor-allocation approaches are appropriate to be used in the engagement models.

The purpose of this report is to address this issue by demonstrating the sensitivity of architecture performance studies to different approaches for interceptor-assignment. In particular, the interceptor allocation strategies found in some of the popular engagement models, including those used during the System Analyses and Key Trade-off (SAKT) Studies (often known as the "horserace" studies) (Ref. 1), are examined and compared to the more sophisticated allocation algorithms currently under development within the SDIO's algorithm development programs. It is shown that in many plausible scenarios the simplistic approaches in the engagement models can lead to predictions of architecture performance which are a factor of two worse (in terms of RV leakage) than those obtained using more sophisticated algorithms. On the other hand, there are equally plausible scenarios for which the simplistic algorithms lead to predictions of architecture performance comparable to those from the sophisticated algorithms. The differences, and in which scenarios they might occur, can be easily understood and will be discussed throughout this report.



This paper is the first of three on the topic of interceptor allocation in strategic defense. This first paper is concerned primarily with the allocation strategy for the defense in an environment in which it has perfect, global information. It will be shown that architecture performance is strongly dependent on the allocation strategy used by the defense and that the "best" strategy is highly scenario dependent. Part 2 of this series contains a discussion on why the information possessed by the defense will realistically contain errors and the degree to which the information may be corrupted. Part 3 of this series addresses the issue of interceptor allocation in the realistic event that the defense does not possess perfect information, e.g., missile-typing may be imperfect, discrimination is imperfect, etc. In this third paper, it will be shown that the allocation strategies that are favored in the present paper provide a degree of robustness, when the defense is working with corrupted information, which is absent with some of the simpler algorithms.

This work was motivated initially by the observation that certain interceptor allocation strategies which worked well in certain scenarios result in degraded architecture performance relative to other strategies in other, equally plausible, scenarios. For example, a scheme for allocating Space-Based Interceptors (SBI) to enemy missiles in their boost phase which assigns a single SBI to a target might work well when the interceptor's probability of kill ( $P_k$ ) are very high ( $> 0.9$ ), but might perform poorly compared to one which allows salvos to be fired when more realistic  $P_k$ 's ( $\leq 0.7$ ) are in effect. That is, salvos might be required in order to ensure that a target is successfully engaged in situations in which the  $P_k$  is low. Unless the firing strategy is flexible enough to allow this, the absolute best that the defense can perform in the boost phase is to have an expected attrition equal to  $P_k$ , which is not tolerable in the advanced architectures where the leakage requirements are on the order of less than 10 percent per phase. Likewise, these "one interceptor per target" strategies will fail in the cases where the defense wishes to ensure survival of selected assets, e.g., in the case of a limited protection system.

Another example is that of a Space-Based Laser (SBL) with a battle manager whose boost-phase goal is to kill as many RVs as possible at each instant without regard to the future, i.e., to maximize its *current* kill rate, a commonly used strategy (Ref. 1). This scheme might perform fairly well in the event that the laser is weak, where weak is defined to mean that the dwell time is long and thus only a small fraction of the threat can be destroyed. This is because the laser would search out the high-value targets, and continually concentrate its firepower on those. However, when the laser brightness is increased, it is easy to envision a scenario in which the defense, using this strategy, fires

first at all of the SS-18s (because they contain the highest number of RVs) and destroys all of them long before they burn out. By this time, however, the faster burning SS-24s and -25s could be burned out and no longer available as targets and thus never engaged. If the defense had chosen a different strategy, in which the SS-24s and -25s are engaged first, and then during the remaining time in which the SS-18s were still burning it engages these, the defense could possibly perform much better. In other words, the ideal battle plan is not decided *a priori*, but will be implemented once the battle manager can assess the performance parameters of the weapons in its arsenal as well as the size, value structure, and characteristics of the threat.

A third example of the need for adaptive battle management tactics occurs when utilizing an ERIS-type interceptor, which has a large footprint that shrinks with time. One strategy embedded in some of the engagement models described in Ref. 1 was to employ a battle manager which preferentially allocates the ERIS interceptors from farms which are closest to the aimpoints of incoming RVs against those RVs. Another was to preferentially allocate the ERIS interceptors which could achieve the earliest intercept. These schemes may work well in the event that the attack is uniformly spread across the continental United States (CONUS), but they can result in degraded architecture performance if the attack is concentrated. For example, if the attack is concentrated on the West Coast, an ideal strategy might be to expend the interceptors from central CONUS on the first shot, saving those on the West Coast for the second and maybe third shots. However, the algorithms in use in most engagement models would expend the West Coast ERIS interceptors first, leaving very few interceptors for a second or possibly third exoatmospheric shot.

There appear to be several factors in a Strategic Defense System (SDS) architecture that push the system into regimes where inflexible allocation schemes and/or conventional rules of engagement can break down. Two of these are the interceptor-rich/interceptor-poor regimes (equivalent to the high-brightness/low-brightness laser regimes or the concentrated/uniform attack laydown) and the high-Pk/low-Pk regimes. Typically, most interceptor-allocation schemes in use in the engagement models contain logic and rules of engagement which work well in the interceptor-poor, high-Pk regime, as evidenced by the allocation algorithms which rarely salvo interceptors. It is possible, however, as shown in Chapter II, to implement flexible rules which work well in all regimes. This is necessary if the scheme is to be used in trade-off studies which span all of these regimes, or else the results can be biased in certain regimes, as we will show in Chapter III. First, however, these two critical factors are examined in more detail.

## A. INTERCEPTOR TO TARGET RATIO

The fact that the defense is interceptor-rich or interceptor-poor, either globally or locally, should radically alter its choice of interceptor-allocation strategies. For example, consider the late midcourse phase of the battle when RVs are coming toward CONUS. If the defense is severely interceptor-poor, it must choose to defend only a select few sites. These sites cannot be determined until the defense has had an opportunity to assess the threat and the predicted aimpoints of the RVs, because the defense must choose to defend only those sites that it has a good chance of saving. This is determined by the number of threat objects (RVs and decoys) aimed at a site, the quality of the available discrimination information, the number of interceptors available to defend that site, and the expected probability of a successful engagement.

Conversely, if the defense is interceptor-rich, it may operate subtractively and attempt to destroy all of the RVs without regard to the predicted impact point. However, in this case, there are any number of ways it may choose to act. For example, consider a defense with three identical interceptors and two targets of equal value. Assume the defense has only one firing opportunity at the targets, for example, as in the boost phase. In almost any battle, a region in which the defense is interceptor-rich will exist. In most of the models described in Ref. 1, the defense will fire one interceptor at each target and hold the third interceptor in reserve, often even if it is leaving the battlespace. As is shown in Chapters II and III, the defense will often be able to increase its effectiveness significantly if it is allowed the flexibility to fire the third interceptor as part of a salvo against one of the targets.

## B. Pk VALUE

As stated earlier, most interceptor allocation strategies in the popular engagement models in use today contain rules of engagement which are based upon the underlying assumption that the defense is working with interceptors having high Pk's. This is evidenced by the fact that virtually all strategies for weapon allocation allow only one shot at a time to be fired at a target. In other words, most schemes do not allow for salvo interceptor assignments. This can often lead to suboptimal performance in an architecture as shown next.

Consider two identical interceptors and two targets of value 10 and 1 point each. Most schemes in the engagement models of Ref. 1 will conduct the engagement so that a single interceptor is fired at each target, regardless of the value of the interceptor's Pk.

This strategy is only optimal, in an average sense, if the  $P_k$  of the interceptor is greater than 0.9. For any value less than 0.9, the defense will do better, on average, to fire both interceptors at the 10-point target. (Depending upon the defense's degree of risk aversity, it may choose the salvo strategy even for  $P_k$  values greater than 0.9; see Chapter V.) This point will be explored further in Chapters II and III when we examine allocation algorithms and insert them into models of typical scenarios.

### C. THE NEED FOR FLEXIBLE ALLOCATION STRATEGIES

The examples described above demonstrate that adaptive, flexible strategies for interceptor allocation should be used by the defense in all phases of the battle in order to ensure a robust performance. They also must be implemented in the models used in analyses, if one desires predictions about architecture performance which are not artifacts of the algorithms embedded in the models. Although it has been shown that a truly optimal interceptor-to-target assignment solution is NP-complete (Ref. 2), near-optimal, yet robust strategies do exist that are useful in a wide range of scenarios and are relatively easy to implement (Refs. 3, 4, 5). In the next section one such adaptive, flexible strategy is discussed for applications to the boost and post-boost phase of the battle. We do not propose it as an algorithm actually to be implemented, but rather, we introduce it because it has many of the desirable features found in the algorithms being developed under the SDIO's algorithm programs. In Chapter III, this approach will be compared in several typical trade-off studies with the approaches taken in the engagement models and it will be suggested that previous predictions about architecture performance in the far-term, low-leakage architecture may have been overly pessimistic.

## II. AN ADAPTIVE APPROACH FOR ALLOCATING INTERCEPTORS TO TARGETS

In this chapter, a flexible strategy is described. It is flexible in the sense that the battle manager must assess the characteristics of the threat and then base the firing decision upon the expected return for each shot. Thus, if the  $P_k$  is fairly low, the defense will salvo interceptors at the high value targets, while neglecting the lower value ones. If the  $P_k$  is high, the battle manager will fire at more of the low value targets and reduce the number of salvos. For all  $P_k$  values in between, the defense will make a smooth transition from the heavy-salvo regime to the single-shot regime, providing the optimal solution at all values of  $P_k$ . Also, if the defense has "extra" interceptors, either locally or globally, they can be expended, as part of a salvo, as long as the expected return on the shot is greater than a user-supplied value.

As demonstrated in the simple example of Chapter I, the defense's "best" strategy is a function of the expected value of single-shot  $P_k$ , the number of available interceptors, and the value structure of the target set. This is demonstrated in Fig. 1, in which a simple scenario is considered. The defense consists of three identical interceptors, and the offense consists of two targets, one worth 10 points and the other worth 2 points. In Fig. 1, the expected number of points killed is shown for three different strategies which the defense might choose. In the first strategy, a "one-interceptor-per-target" approach is taken, and thus, two interceptors are fired, one at each target. In the second, the defense is limited to fire two of its interceptors, but elects to salvo its interceptors at the higher value target. Finally, in the third, the defense is allowed to fire all three of its interceptors, if the expected return for firing an interceptor is greater than, or equal to, 1 point per interceptor. (In this case, the only time all three interceptors are not fired is when the  $P_k$  is equal to 1.0.)

As demonstrated in Fig. 1, the strategy in which all three interceptors are fired is always as good, or better, than the two others, as one would expect. However, between the other two strategies, it is seen that for lower values of the  $P_k$ , the salvo strategy is superior, while for higher values of the  $P_k$ , the one-interceptor-per-target strategy is better.

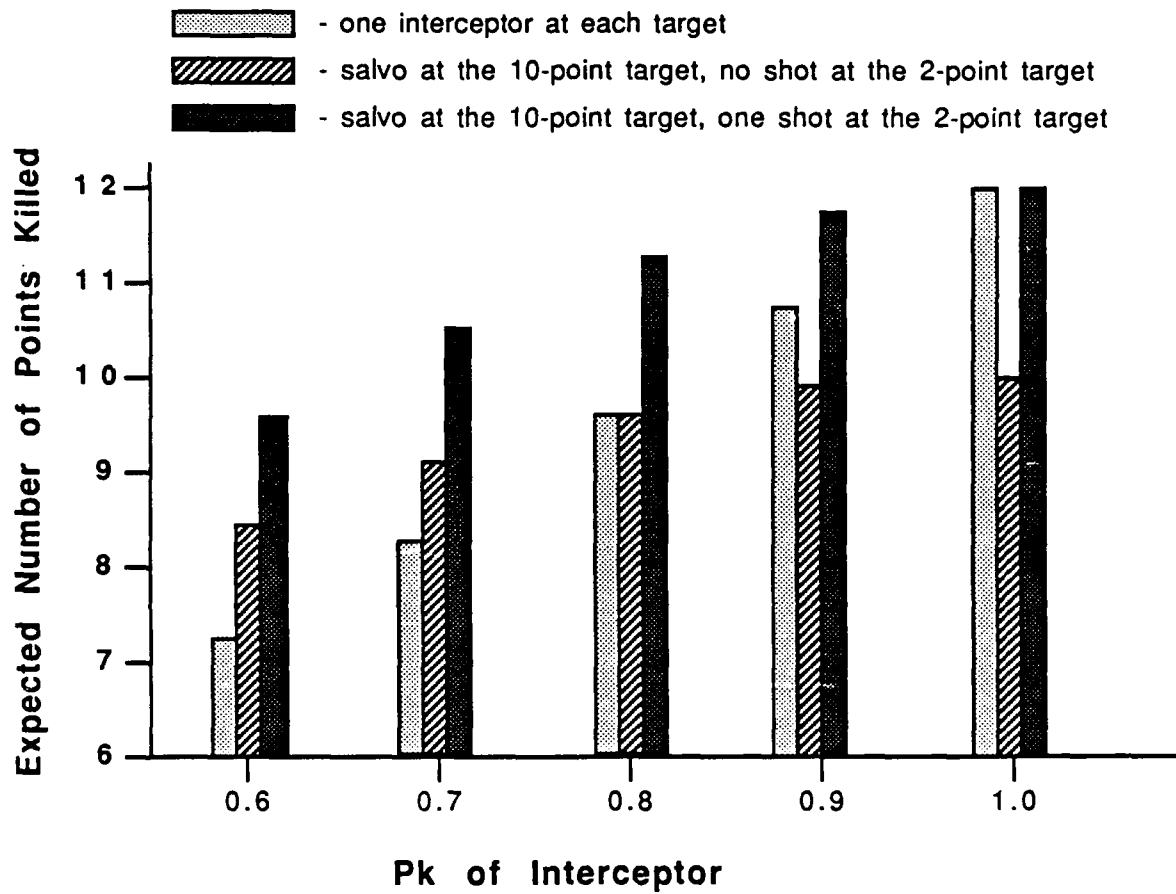


FIGURE 1. Expected number of points destroyed for a defense consisting of three identical interceptors and two targets. The targets are of value 10 and 2 points.

Thus, when designing an algorithm for use in the SDS, or in a simulation of the SDS, an approach is desired in which the salvo becomes the preferred tactic in interceptor-rich/low- $P_k$  regimes, while the one-interceptor-per-target strategy is preferred in the interceptor-poor/high- $P_k$  regimes.

The fact that a salvo is desirable in many cases is a reflection of the fact that the interceptor assignment problem is inherently nonlinear. By this, we mean that the return from each extra shot at a target, in a single firing opportunity, is less than the shot before it. For example, in Fig. 2, the returns for a scenario in which there are 40 equal value targets, and 120 equal interceptors are plotted, as a function of the number of interceptors fired. This figure serves to illustrate graphically that the returns are not linear, but rather decrease with the number of shots taken.

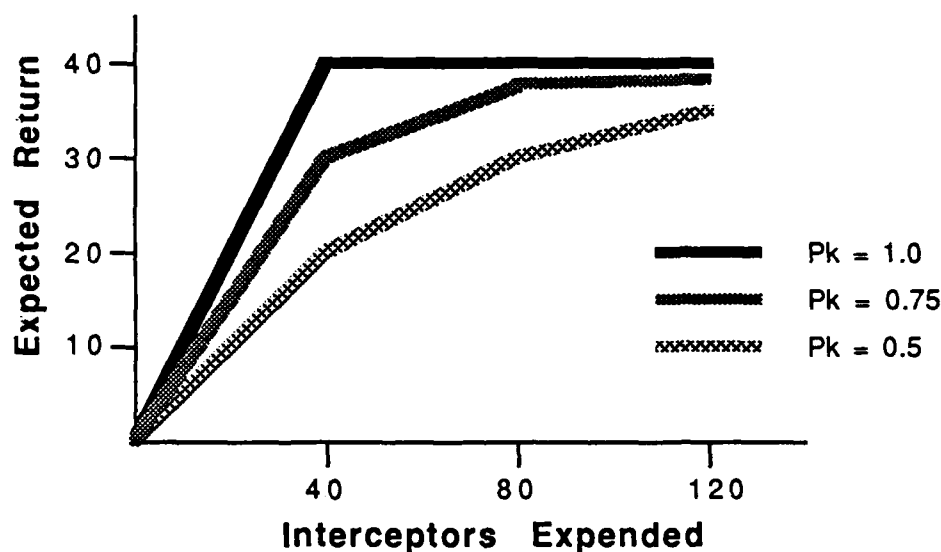


FIGURE 2. Expected return, in terms of targets destroyed, for a salvo size of 1 (40 interceptors expended), 2 (80 interceptors expended), or 3 (120 interceptors expended) for a 40-target threat.

The fact that the returns on interceptor allocation are nonlinear has led to the use of nonlinear approaches by many of those developing allocation algorithms under the SDIO's programs. In order to set up one rather general formulation, we propose a scenario consisting of a simultaneous launch of missiles. After an appropriate time delay for system enablement, track formation, missile typing, and interceptor assignment, the defense can commence firing its interceptors. The value which the defense assigns to each intercept is

equal to the number of RVs expected to be on the target at the time of intercept. For example, a missile which contains ten RVs will have a value of 10 throughout its boost phase, while in the busing phase the value of the intercept will depend upon the time of interceptor arrival, i.e., those intercepts occurring early in the post-boost phase will have a higher value than those occurring later in the post-boost phase.

A formulation of this type leads naturally to the concept of the "engagement matrix." This matrix represents, at any moment in time, which SBIs, if committed at that moment, can engage which targets. A typical engagement matrix for three carrier vehicles (CV) and four boosters, might appear as in Fig. 3. Booster #1 might represent a missile capable of carrying 10 RVs, and thus, the value of 10 in the (1,1) element implies that an SBI from CV #1 can intercept that missile in its boost phase. Conversely, an SBI from CV #3 cannot intercept it at all and an SBI from CV #2 can reach this missile only very late in its busing phase, when only a single RV remains on the bus.

		Missile			
		1	2	3	4
Carrier Vehicle	1	10.0	2.0	2.0	0.0
	2	1.0	2.0	1.0	1.0
	3	0.0	7.0	1.0	7.0

**FIGURE 3. Illustrative values for the engagement matrix for four missiles and three CVs. Interceptors from different CVs may achieve different numbers of RVs killed at the same booster due to their ability to engage at different times in the boost or post-boost phase.**

One approach for obtaining an optimal strategy in this scenario is to use a nonlinear objective function (Ref. 6) which can be formulated as follows. For any pairing of CV  $i$  and target  $j$ , one must consider the probability that target  $j$  is alive when the SBI from CV  $i$  is at the intercept point. This is given by the cumulative probability that target  $j$  survives all interceptors assigned to it with earlier intercept times. This can be expressed as



$$Q_{ij} = \prod_{k \in S_{ij}} (1 - P_{kj})^{X_{kj}}$$

where  $S_{ij}$  represents the set of all interceptors allocated from the CVs which can reach target  $j$  before the interceptor(s) from CV  $i$ . The  $X_{kj}$ 's represent the number of interceptors allocated from CV  $k$  to target  $j$  and  $P_{kj}$  is the probability of kill for an interceptor allocated from CV  $k$  to target  $j$ . Given this, the defense, wishing to maximize the RV destruction through the post-boost phase, would formulate its objective as

$$\text{MAX} \sum_{i,j} Q_{ij} T_{ij} [1 - (1 - P_{ij})^{X_{ij}}]$$

where

$T_{ij}$  = value of target  $j$  for an intercept with an SBI from CV  $i$ ,

$[1 - (1 - P_{ij})^{X_{ij}}]$  = probability that target  $j$  is destroyed by the interceptors assigned to it from CV  $i$ ,

$i \leq 1 \leq \text{number of CVs in the battle, and}$

$j \leq 1 \leq \text{number of boosters (buses).}$

The solution, the  $X_{ij}$ 's, which represent the number of SBIs from CV  $i$  fired at missile  $j$ , will provide the minimum number of RVs surviving through the boost/post-boost phases of the battle, providing the value of  $T_{ij}$  is the number of RVs on the target at the time of intercept.

In applying this formulation to interceptor assignment, one wishes to constrain this problem to integer solutions (the  $X_{ij}$  are integers). It has been shown (Ref. 2) that the problem is NP-complete and does not lend itself to any convenient solution techniques. In order to make the problem more tractable, we make the approximation that the single-shot  $P_k$  is a constant, and independent of the target and the interceptor. Also, we set an upper limit on the value of  $X_{ij}$ . This allows us to arrive at an exact solution to our approximate problem, by a technique described next.

In order to introduce the nonlinearities into the problem, we expand the engagement matrix to any degree desired, to reflect the fact that if the  $P_k$  is less than 1.0, there is value to an extra shot in a salvo. For example, if the  $P_k$  of the interceptor were 0.7, and a maximum of three shots at any target is desired, the engagement matrix of Fig. 3 could be expanded to appear as shown in Fig. 4, where fictitious targets of lesser value are collocated with the real target. These "ghost" targets represent the expected extra return, in

terms of extra RVs destroyed, for firing salvos at any given missile. Now the problem appears to be one of twelve targets and three CVs, and each target can be fired on once, at most, depending upon the available SBI inventory. For example, from the expanded matrix it is easy to see that there is a better return to firing two SBIs from CV #1 at missile #1 (expected return is 9.1 RVs) as opposed to firing one interceptor each at missiles #1 and #2 (expected return is 8.4 RVs).

		Missile											
Carrier	Vehicle	1A	1B	1C	2A	2B	2C	3A	3B	3C	4A	4B	4C
	1	7.0	2.10	0.63	1.4	0.42	0.13	1.4	0.42	0.13	0.0	0.00	0.00
	2	0.7	0.21	0.06	1.4	0.42	0.13	0.7	0.21	0.06	0.7	0.21	0.06
	3	0.0	0.00	0.00	4.9	1.47	0.44	0.7	0.21	0.06	4.9	1.47	0.44

FIGURE 4. The engagement matrix of Fig. 3, expanded to include the "ghost" targets. Ghost targets are shaded gray. A Pk of 0.7 was assumed for all intercepts. The maximum size of the salvo was set to be three interceptors.

It should also be noted that in order to avoid wasting interceptors on the very low-value shots, such as those with expected returns of 0.06 RVs in the matrix of Fig. 4, one can set to 0.0 any matrix element which is below a minimum return that the defense is willing to accept for an expended SBI. For example, if the defense wished to expend interceptors only if the return were greater than 0.5 RVs, then sixteen of the non-zero matrix elements in Fig. 4 would be set to zero and no interceptors would then be allocated.

Once the problem is formulated in this manner, it can be solved by common techniques of linear programming. In the linear programming technique, again applied to a boost/post-boost phase scenario, the defense would choose to maximize the number of RVs destroyed. This can be expressed as

$$\text{MAX} \sum_{i,j} T_{ij} X_{ij} P$$

subject to the constraints of

$$\sum_j X_{ij} \leq \text{Number of interceptors on CV } i$$

and

$$\sum_i X_{ij} \leq 1$$

where

$X_{ij}$  = number of interceptors fired from CV  $i$  to target  $j$  ( $= 0$  or  $1$ ). (Here, target  $j$  can be the original or the ghost.),

$T_{ij}$  = value of the intercept from CV  $i$  to target  $j$ , and

$P$  = single-shot  $P_k$  for all shots.

Algorithms for solution of the linear programming problem can be found in any standard textbook on the subject (Ref. 7) or in documents which examine this explicit problem (Ref. 8). We note that the approach taken here is not applicable for a  $P_k$  which varies for each interceptor-target pair. At the best, it works if the  $P_k$  is a function of only the target, independent of which interceptor is committed to it. Since we are using this simplified approach to illustrate the utility of nonlinear algorithms, however, it represents no real drawback. There are nonlinear approaches which can incorporate a  $P_{ij}$  (Ref. 3)--we choose not to use them in these simplistic examples.

The utility of this method, and other nonlinear approaches, is that the interceptor allocations which result are flexible in the sense that as the  $P_k$  falls, the defense will begin to salvo more often. If the  $P_k$  equals 1.0, the solution will collapse to the standard linear programming solution on the engagement matrix of Fig. 3 and no salvos will be fired. The adaptiveness of this method is not in the algorithm--standard linear programming techniques are used--but, rather, in the generation of the engagement matrix which incorporates the ghost targets.

In the following chapter, the system performance using this method to assign interceptors to targets will be compared to system performance in which rigid algorithms are used under the four combinations of interceptor-rich/interceptor-poor and high- $P_k$ /low- $P_k$ , and the robustness and utility of this new technique will be illustrated.

However, in order to illustrate the method in some detail, the interceptor allocations are presented for the engagement matrix of Fig. 3 for the case in which each CV has two interceptors. These solutions are optimal, in terms of the number of RVs destroyed. It should be pointed out that they are not unique. The allocations are presented explicitly in Fig. 5. Several features can be seen immediately.

First, the adaptive, nonlinear technique outperforms the conventional linear approach by destroying 1.8 extra RVs on average. This was achieved by firing two extra SBIs which are not fired in the usual linear approach. It is impossible under a linear formulation to fire the two SBIs from CV-2 because of the constraints that only a single interceptor be fired at a single target. It is important to note that even if the defense had been constrained to fire four SBIs under the adaptive solution, it still would have outperformed the linear programming solution (18.9 to 18.2 RVs killed) by assigning a salvo of two at the 10-point booster, booster #1, rather than having CV-1 fire one shot at booster #1 and one shot at the 2-point booster, booster #3. It is exactly in cases like this that an algorithm is needed which reflects the parameters of the weapon system and the value structure of the threat.

Finally, it should be pointed out that in the adaptive technique, the defense can choose to fire only those interceptors that would achieve some minimum return, on average. For example, in the firing strategy for the adaptive algorithm, the minimum allowable expected return was arbitrarily set to 0.3 RVs per SBI. This value can be changed as the defense sees fit, depending upon the size of the threat relative to the defense, the readiness level, etc.

#### Linear Approach

CV →	Target	Expected RVs Killed
1	1	7.0
1	3	1.4
3	2	4.9
3	4	4.9

**Total: 18.2**

#### Nonlinear Approach

CV →	Target	Expected RVs Killed
1	1A	7.0
1	1B	2.1
2	2B	0.4
2	3A	0.7
3	2A	4.9
3	4A	4.9

**Total: 20.0**

**FIGURE 5.** The SBI allocations for the conventional linear programming solution from the engagement matrix of Fig. 3 (left) and for nonlinear approach using the expanded matrix of Fig. 4 (right). In the second case, the nonlinear approach, if the defense chooses to fire only four interceptors, it will destroy 18.9 RVs, which is still better than the linear approach.

### **III. COMPARISON OF INTERCEPTOR-TO-TARGET ASSIGNMENT ALGORITHMS IN SIMPLE TRADE-OFF STUDIES**

In this chapter the nonlinear approach described in Chapter II is compared to the conventional linear programming and the earliest-intercept approaches found in the popular engagement models. The conventional linear programming approach with missile typing capability was chosen because it represents one of the best efforts seen to date to perform the interceptor allocation function in most simulation models. The "earliest-intercept" algorithm, with or without autonomous CVs, is often used because it is simple to implement, and so in our study we examine both cases. It is first implemented in the form in which each CV acts autonomously, and when presented with a choice of targets, fires at those which will allow the earliest possible intercepts, regardless of target value (i.e., no missile-typing capability) and regardless of what the other CVs are doing. It is also implemented in a form of a coordinated system, in which all of the targets are ranked according to their value (in this case, the expected number of RVs on the booster/bus at time of intercept) and then whatever SBI can achieve the earliest intercept is paired with that target. An approach of the latter type requires a global battle manager, with missile-typing capability. For ease of discussion, the four techniques will be referred to as the nonlinear approach, the linear approach, the coordinated earliest intercept approach, and the autonomous CV approach.

Details of the offensive threat and defensive constellation used in our calculations can be found in the appendix. The model used to perform these calculations is one developed at IDA to simulate the boost and post-boost phases of the battle. The emphasis in the development of the model has been to allow flexibility in the interceptor allocation strategy. To simplify the study and remove emphasis from features not of interest in this study, it was assumed that the defense has sensors which provide global coverage with infinite resolution, which will provide sufficient information to allow perfect track formation and perfect missile typing. Likewise, all communications were made perfect so that all battle managers have the information necessary for making allocation decisions.

(These constraints will be relaxed in papers 2 and 3 of this series.) The model makes use of threat tubes to simplify the programming and keep the run-time low. All leakages reported in this chapter are expected values. (In Chapter IV the problems which can arise from using expected values are discussed in terms of the risk-averse nature of the defense in the strategic defense scenario.)

The first trade study we performed was one of the simplest imaginable--leakage through the post-boost phase as a function of the number of SBIs. (It should be noted that we choose to fix the number of CVs at 360 and vary the number of SBIs/CV. Certainly the weapon-rich scenarios with 50 SBIs/CV are not necessarily reasonable, but they should not detract from the validity of our calculations and results.) The results are shown in Figs. 6 and 7 for interceptor Pk-values of 0.9 and 0.6, respectively. It was found that 45 of the 360 CVs were capable of participating in the battle. The scenarios in these two figures span the four regimes discussed in the introduction, interceptor-rich/interceptor-poor and high-Pk/low-Pk. In the nonlinear approach, the minimum acceptable return for an SBI was set to 0.09 and 0.24 RVs destroyed for the high- and low-Pk cases, respectively, which allowed the defense to salvo at least two SBIs at any booster, provided inventory was available. (In other words, by setting a value for the minimum acceptable return (expected number of RVs destroyed) for each expended SBI, we are limiting the maximum salvo size. This is equivalent to setting a limit to the number of ghost targets in the expanded matrix.) The salient features of these simple trade-off studies will be discussed next.

First, it is important to note that the nonlinear approach outperforms the other three techniques for both values of Pk at all levels of defensive inventory, showing its robustness in all regimes. Next, note that the linear approach provides a solution which is closer to the nonlinear solution in the high-Pk regime, than in the low-Pk regimes. This should be expected, given that the nonlinear solution will collapse to the linear programming solution for a Pk of 1.0. Another noteworthy feature occurs in the interceptor-rich regime (18,000 SBIs). Here the linear solution results in 70 percent more leakage than that resulting from the nonlinear solution for the lower value of Pk, as shown in Fig. 7. Even at the high value of Pk (Fig. 6), the linear solution results in about 50 percent more leakage than the nonlinear approach. This demonstrates the need for an adaptive, nonlinear approach whenever the defense is interceptor-rich.

In the interceptor-poor regime (10-20 KKV/CV), it is seen from Figs. 6 and 7 that the linear and the nonlinear approaches perform similarly. This is to be expected because the defense does not possess enough SBIs to shift them from the low-value targets

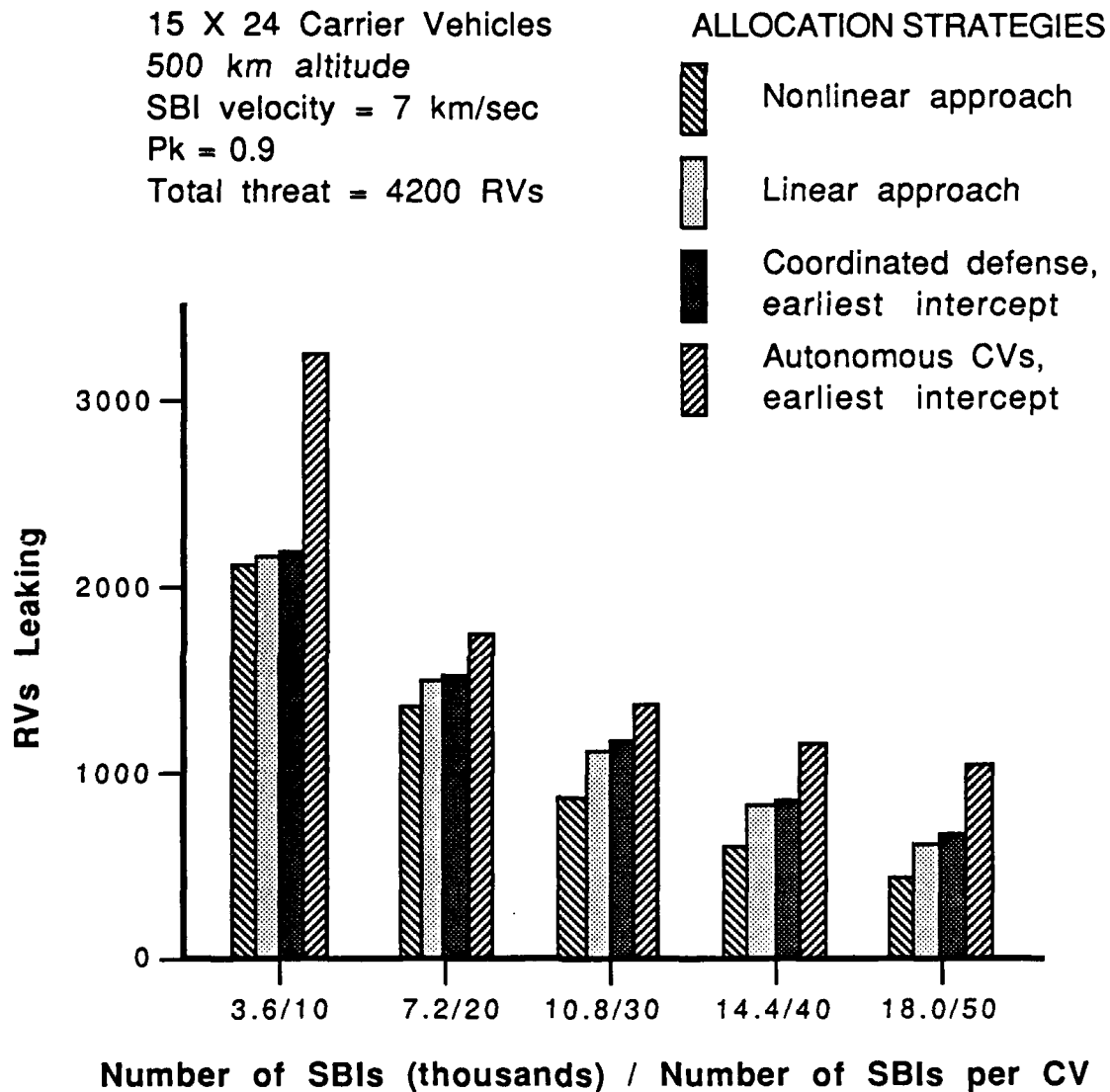


FIGURE 6. Leakage through the post-boost phase as a function of the number of SBIs for the four allocation schemes discussed in the text. The  $P_k$  for all intercepts is 0.9.

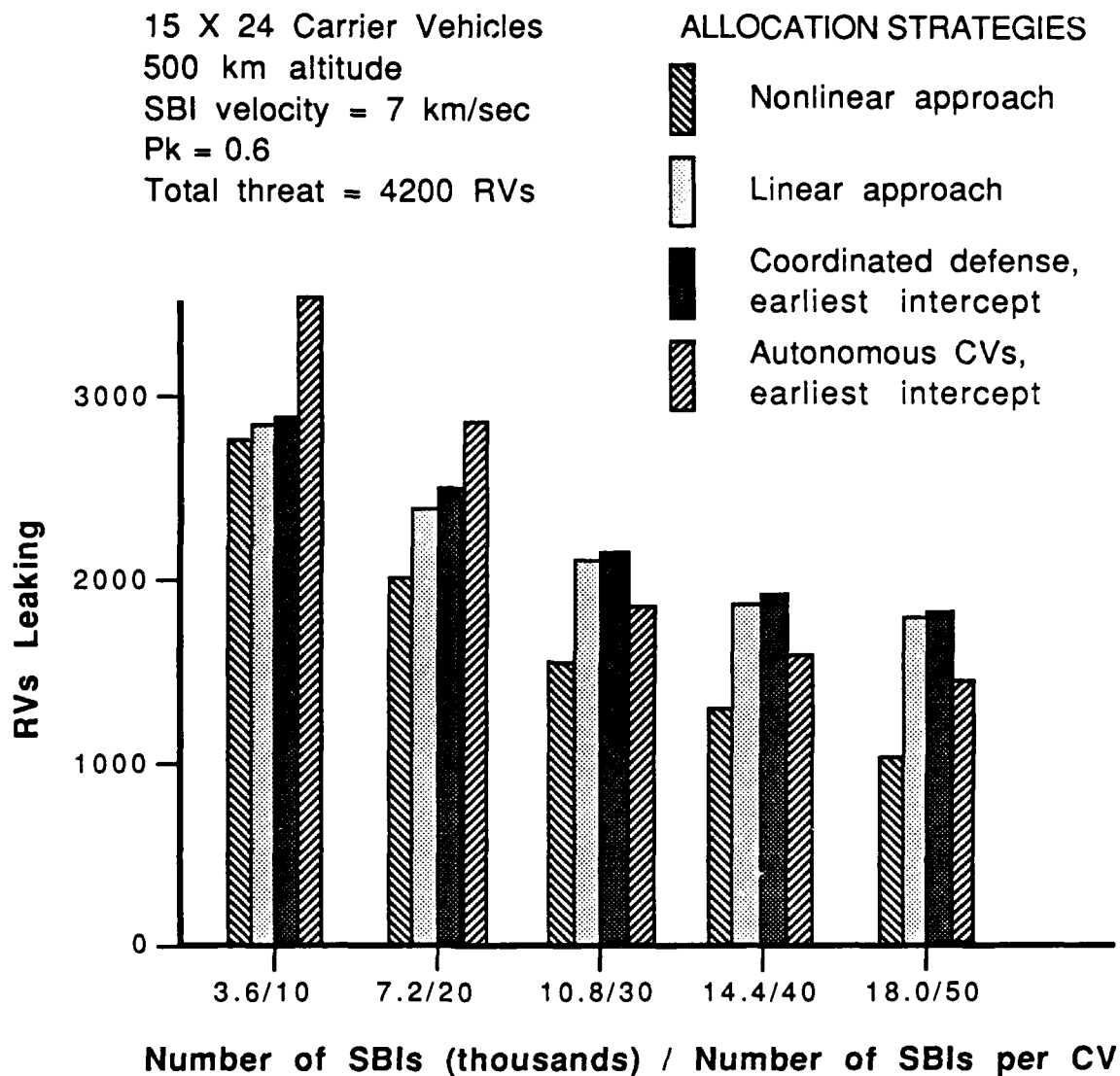


FIGURE 7. Leakage through the post-boost phase as a function of the number of SBIs for the four allocation schemes discussed in the text. The  $P_k$  for all intercepts was 0.6.



to the high-value targets, nor to fire many salvos. Rather, in both cases the defense is firing virtually all of its available SBIs at the high-value targets in salvos of one.

The performance of the autonomous-CV defense is quite interesting. In the interceptor-poor regime, it performs significantly worse than the other three techniques for both values of  $P_k$ . This is because the CVs often fire their weapons at the same targets, due to lack of coordination among themselves along with the fact that the CVs do not select the high value targets to engage preferentially. In the weapon-rich regime, however, it is interesting to note that in the case of the low  $P_k$ , the autonomous-CV approach outperforms the linear and earliest intercept approaches. This is a result of the fact that when each CV releases its weapons (in all, 45 CVs were found capable of participating in the battle) in the low- $P_k$  case, the extra SBIs are not really wasted, because they can make up for what some of the other SBIs missed. In other words, the net result is a salvo, which is very useful in the low- $P_k$ /interceptor-rich regime, as discussed earlier. In the case of the high value of  $P_k$ , however, the autonomous-CV approach did not surpass the linear scheme, because the value of the  $P_k$  was not the overriding factor--90 percent of the SBIs in both cases successfully engaged their targets and thus the need for a salvo was not as important.

Several comments about both of the earliest-intercept strategies should be made here. When employed with autonomous CVs, this method of allocating SBIs to CVs is not an efficient way to use the SBIs. Although, as discussed above, it outperformed the linear and coordinated earliest-intercept approaches in the weapon-rich, low- $P_k$  regime, the price was extremely high in terms of the number of SBIs fired--2250 in the autonomous-CV case versus 650 in the linear approach. Likewise, an architecture based upon this BM/C3 concept is easily spoofable. For example, the offense, by launching ten ICBMs, one every six seconds, could draw down about 6-8 percent of the CVs (21-26 in this case) in one minute because each CV which can engage the ICBMs will do so, regardless of the fact that over twenty other CVs are also doing the same. This obviates the need for an ASAT attack on the part of the offense.

We turn next to a more detailed discussion of the "price" paid, in terms of expended interceptors, for the increased system performance in a nonlinear approach. It is sometimes possible, as described in the example shown in Fig. 5, to obtain better performance for free, simply by shifting interceptors from low-value targets into salvos at higher-valued ones. However, in the usual implementation of the nonlinear technique, the defense will fire more interceptors than in the linear approach during each firing opportunity. It has not yet been determined when the defense should stop firing in the boost/post-boost phase and

allow the midcourse interceptors to engage the remaining threat. We suggest that the value of this threshold should be set in the neighborhood of 0.5 RVs per SBI expended. This is based on the fact that the midcourse "effective  $P_k$ " will be substantially less than the boost/post-boost phase  $P_k$  due to the proliferation of decoys and the fact that an RV is a smaller target than the booster or bus. Thus, if the defense can commit an interceptor with an expected return of 0.5 RVs destroyed (not to mention tens to hundreds of decoys which may be on the booster or bus) in the boost or post-boost phase, it should probably do so.

In order to place the results from the nonlinear approach of Fig. 7 on a more even footing with those of the linear approach, then the minimum return per SBI fired in our nonlinear approach can be set to be the value of  $P_k$ . (Recall, in Figs. 6 and 7, the return was set at  $P_k(1-P_k)$ .) This is exactly the same minimum return obtained in the linear approach, which arises when an interceptor is allocated to a 1-RV booster. If this is done, then the leakages from the nonlinear approach shown in Fig. 7 do not change significantly. The new curves are shown in Fig. 8, where the leakage is plotted for a  $P_k$  of 0.6. Along with this is the number of SBIs fired to achieve these results. It is noteworthy that no SBI is committed in this scenario, regardless of allocation strategy, whose expected return is less than 0.6 RVs destroyed. This could be better than can one might expect to obtain in midcourse, given that the presence of decoys will lower the "effective  $P_k$ ," due to the uncertainty in the target set.

Several interesting features can be seen from an examination of Fig. 8. First, the linear and nonlinear approaches are seen to be similar in the interceptor-poor regime (3600 SBIs) as would be expected from the earlier discussion in Chapter II. It is also noted that the expected number of RVs killed per SBI fired in Fig. 8 can be *greater* in the linear approach than in the nonlinear case. For example, in the interceptor-rich case, the average return per SBI expended is about 3.5 RVs in the linear approach, while for the nonlinear approach it is only about 2.5 RVs. This is simply the result of more interceptors being expended in the nonlinear case than in the linear case. As suggested earlier, however, the defense *ought* to continue to expend its SBIs until it expects a return of less than roughly 0.5 RVs per expended SBI, provided it has sufficient inventory. In fact, the interceptor-rich defense with 18,000 SBIs, using the nonlinear approach in Fig. 8, fires 450 more SBIs than the defense using a linear approach, but destroys, on average, 700 more RVs, a highly favorable return.

A second simple trade study was performed in order to compare algorithms--leakage through the post-boost phase as a function of the interceptor velocity. The results

of this study are shown in Fig. 9. The system with a high-velocity interceptor corresponds to an interceptor-rich system because significantly more CVs will lie within the battlespace than in the low-velocity interceptor case. Again, the same trends emerge when the results are examined. In the interceptor-rich case (i.e.,  $\Delta v = 9$  km/sec), the adaptive, nonlinear approach outperforms the linear approach for both values of  $P_k$ . In particular, for the lower value of  $P_k$ , the linear approach results in about twice the amount of RV leakage, while at the higher value of  $P_k$  it results in about 70 percent more leakage. Even in the interceptor-poor case, the difference is significant if the  $P_k$  is low, with the linear approach resulting in about 20 percent more leakage over the nonlinear approach. As in the previous examples of Figs. 6 and 7, it is seen that in the interceptor-rich scenarios, the nonlinear technique outperforms the linear ones by a significant amount.

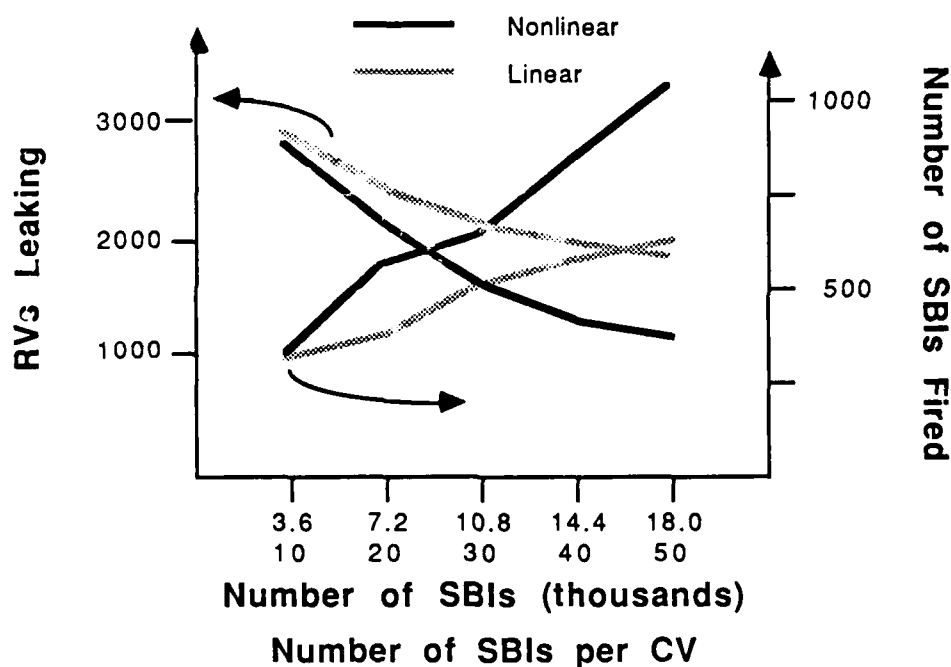


FIGURE 8. The curves which go from upper left to lower right represent the leakage through the post-boost phase as a function of the number of SBIs per CV for the linear (gray lines) and nonlinear (black lines) approaches. The curves which go from lower left to upper right correspond to the number of SBIs committed in the two cases. The  $P_k$  value was 0.6. The minimum allowed return for an expended SBI in the nonlinear allocation was set to 0.6 RVs.

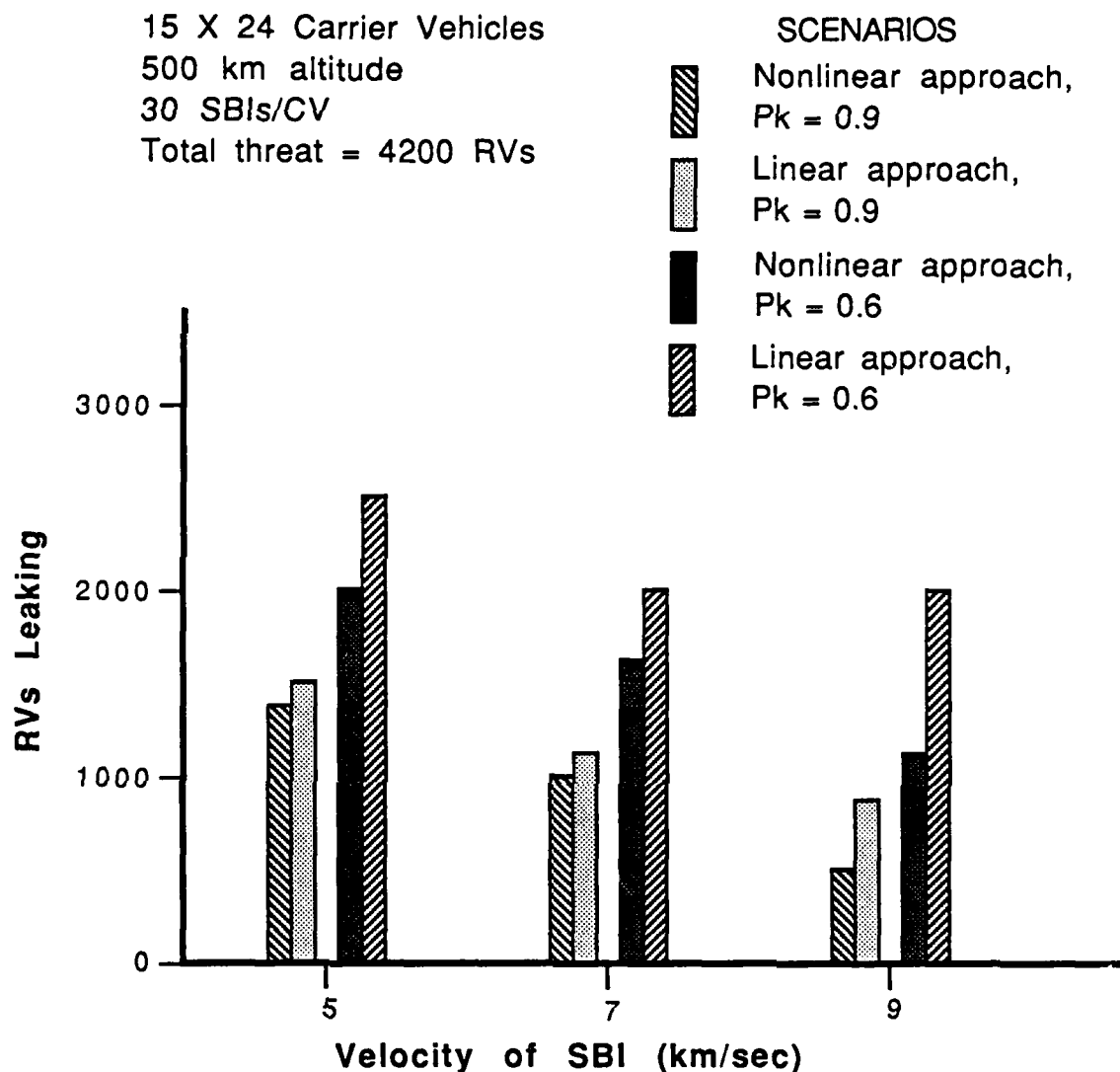


FIGURE 9. Leakage through the post-boost phase as a function of the velocity of the SBI for two of the schemes discussed in the text.

These results from these two simple trade-off studies have served to substantiate the assertion of Chapter I that the same algorithm, unless it has flexibility built into it, should not be used in trade-off studies which span all architecture regimes. This will be discussed again in Chapter V, but first, an important, extra benefit of the nonlinear approach is demonstrated in Chapter IV.

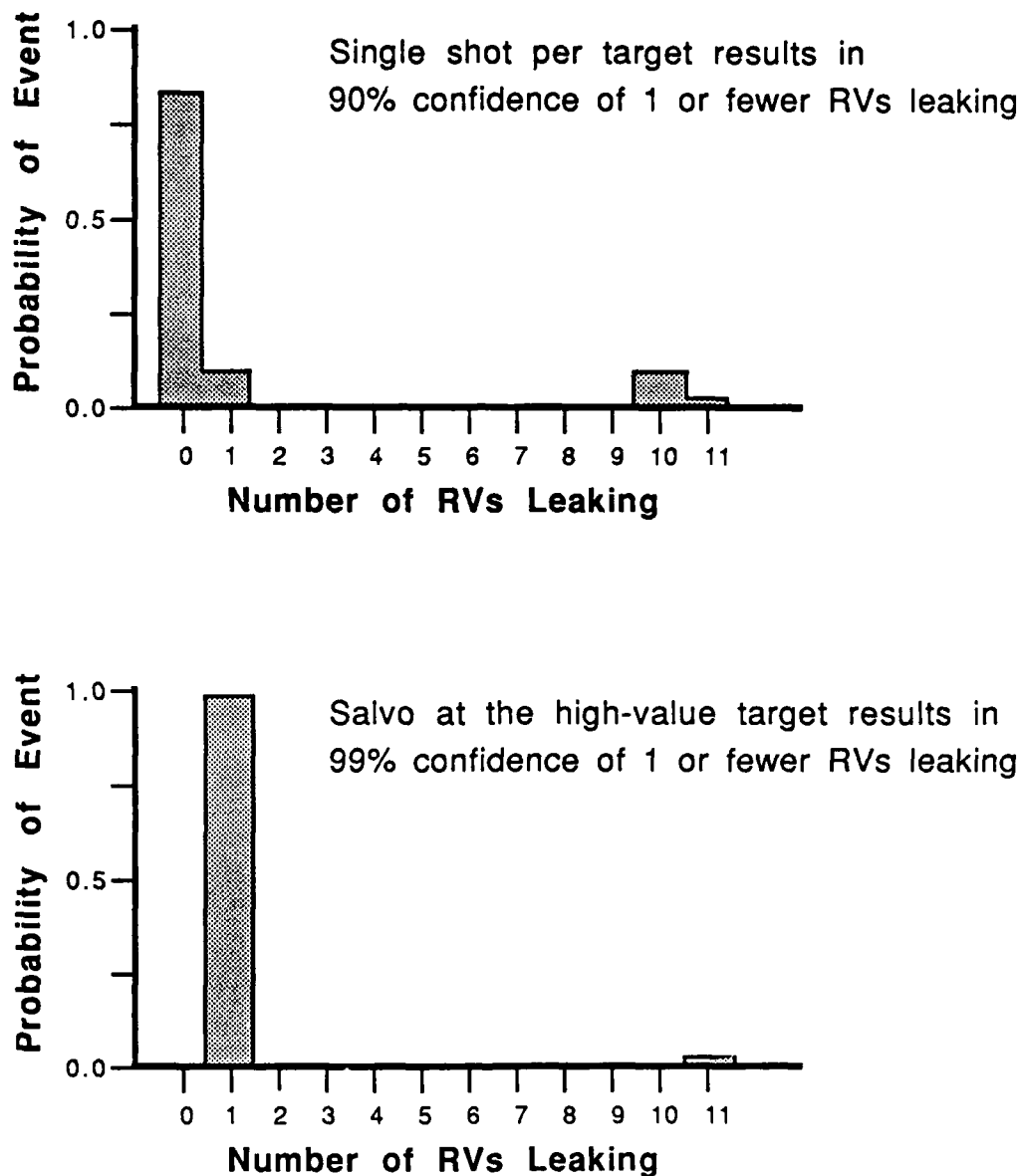
#### IV. RISK AVERSITY OF NONLINEAR APPROACHES

Aside from providing a better solution, on average, to the interceptor allocation problem, the nonlinear strategy provides a solution which is more robust, in the sense that the spread of possible outcomes about the mean value is smaller than that of the linear approach. Thus, if the system is being designed not for the mean value of performance, but for some value in which the defense has, say, 90 percent confidence, the nonlinear approach performs even better than demonstrated earlier.

This is illustrated by a simple example. Consider a defense with two interceptors, both of which have a  $P_k$  of 0.9. The offense has launched two targets, one of value 10 and the other of value 1. For the  $P_k$  of 0.9, it does not matter, on average, whether or not the defense chooses to fire one interceptor per target or both interceptors at the 10-point target--the expected leakage in both cases is 1.1 RVs. However, if all possible outcomes of both strategies are examined, and their probabilities of occurrence, the distributions of Fig. 10 result. It is seen immediately that the salvo strategy will provide a higher confidence that the defense is not overwhelmed with a 10 or 11 RV leakage, resulting from random bad luck with the interceptors. In other words, there is a significant degree of risk aversity built into the salvo strategy, relative to the single-shot strategy. This is especially significant when a multi-layer defense is considered. The defense, in the *early* phases, should assume a risk-averse posture to ensure that the subsequent phases are not overwhelmed as a result of statistical fluctuations. Conversely, in the later phases of the battle, depending upon the defense's mission, the single-shot strategy may be chosen if zero-leakage is the goal.

The ability to choose between the salvo and the single-shot strategies of the above paragraph is exactly the flexibility provided with a nonlinear approach over a linear one. In order to illustrate this on a realistic scenario, the interceptor-poor (3600 SBIs) and interceptor-rich (18,000 SBIs) architectures of the appendix were examined in more detail for a  $P_k$  of 0.8 and an interceptor with an axial velocity of 7 km/sec. In particular, the allocation strategies were computed, and then each individual intercept was examined, by a random number draw, to determine whether or not it was successful. In all,

1000 Monte Carlo trials were performed. The results, shown in Fig. 11, are dramatic in the interceptor-rich case, but only slightly significant in the interceptor-poor case.



**FIGURE 10.** The spread in possible outcomes from two different strategies, which, on average, yield the same results. It is seen that the strategy using the salvo provides greater security against a possible leakage of 10 or 11 RVs. Conversely, the single-shot strategy offers the only chance of destroying all 11 RVs.

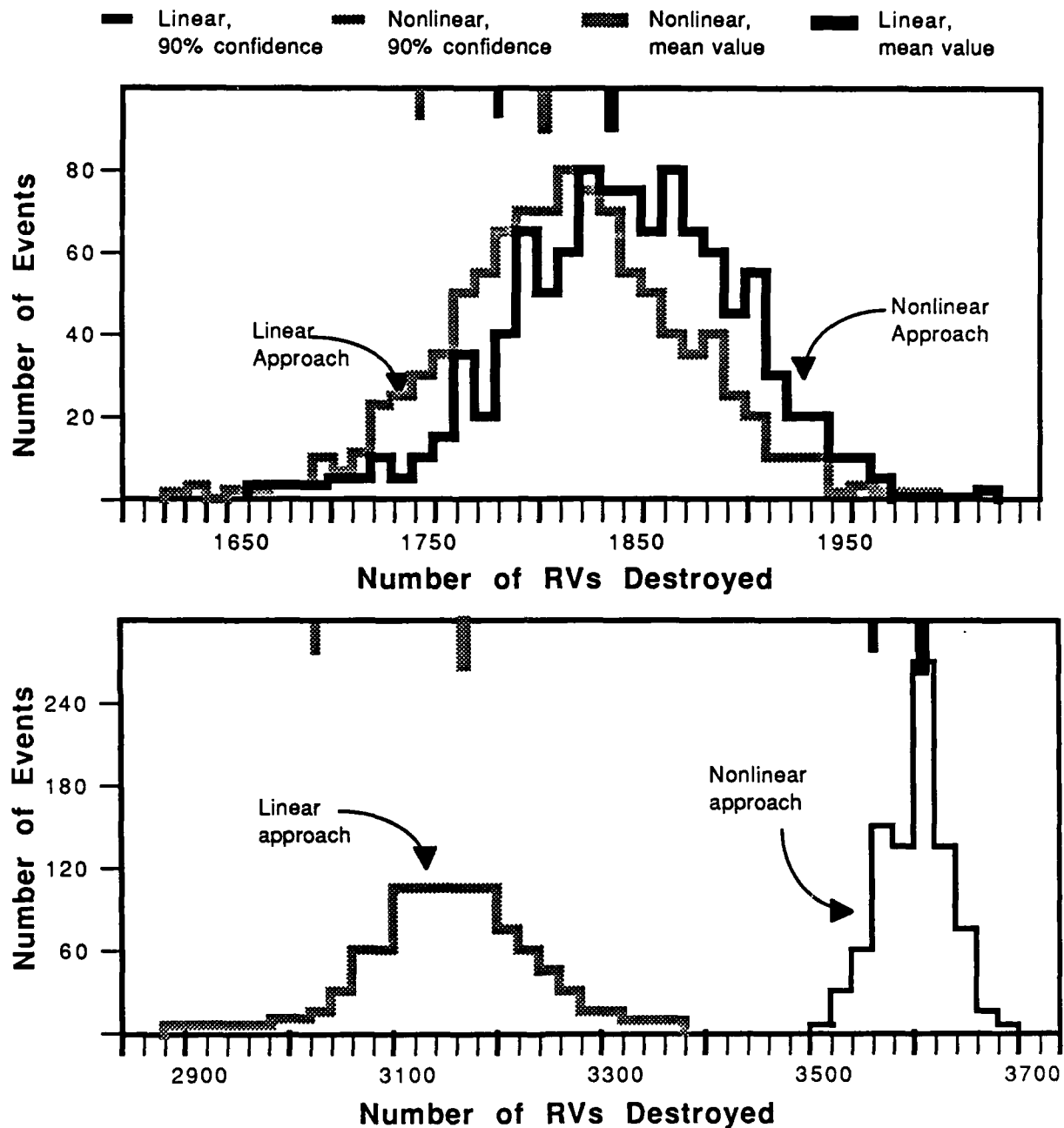


FIGURE 11. The spread of outcomes for 1000 repetitions of two of the architectures described in Chapter II. The upper figure is for the 3600-SBI case and the lower figure is for the 18000-SBI case. The  $P_k$  in both cases was 0.8. Aside from providing a better defense, on average, it is seen that the nonlinear case provides even better protection at the 90 percent confidence level.

In the interceptor-poor architecture, the average leakage values differed by 47 RVs, on average, while the leakage values differed by about 55 RVs at the 90 percent confidence level. This is a rather small difference. However, in the interceptor-rich architecture, the average leakage values differed by 440 RVs, while at the 90 percent confidence level the leakage values differed by 485 RVs. This is especially significant when one considers that aside from the extra 45 RVs which might leak, as many as several thousand decoys may leak into the midcourse phase with them.

In order to illustrate this utility of risk aversity more fully, we will look at the processing capability needed to perform the midcourse tracking function. It is thought that the processing capability will scale as the number of objects raised to the third power (Ref. 6). If this is the case, and the system is sized to track the average number of leakers, then a system employing the nonlinear approach might require  $440^3$  "units" of processing capability, ignoring decoys. Conversely, if the system is sized to track the leakage in 90 percent of the cases, it would require  $485^3$  units of processing capability. This results in a requirement of 1.34  $(= (485/440)^3)$  times the processing capability needed to track the mean number of leakers, if the midcourse tracker is sized to operate for 90 percent of the possible boost/post-boost phase outcomes.

Now, consider the linear approach, and *assume* that enough weaponry has been added to the SBI system so that its average performance equals that of the nonlinear approach (440 RVs leaking). If we assume that the spread about the mean is the same as shown in Fig. 11, then at the 90 percent confidence level the leakage will be about 580 RVs. In this case the midcourse tracking processor would have to be 2.29  $(= (580/440)^3)$  times as large as that sized to handle the average leakage. So, aside from the extra required weaponry, we see severe requirements are placed on the subsequent phases, due to the larger fluctuations about the mean which arise from the linear approach.

Similar conclusions could be drawn concerning the required number of midcourse and/or terminal interceptors needed to ensure that the system must meet its mission with a high degree of confidence. By choosing risk averse strategies in the early phases of the battle, it proves much easier to avoid catastrophic failure of the later phases by random fluctuations in system performance. Likewise, the design of the later phases is simplified, because the designer has much greater confidence in the size of the threat leaking through the earlier phases.



## V. CONCLUSIONS

An adaptive, flexible technique for assigning interceptors to targets has been described and the magnitude of its effects on architecture performance has been quantified in several scenarios. It was shown that a nonlinear approach can provide a high level of architecture performance in many more scenarios than can the more common linear approaches.

In Chapter III it was demonstrated that previous analyses of interceptor-rich architectures (which were the primary focus of the SDIO prior to early 1987) may have been significantly pessimistic in their predictions of boost/post-boost phase performance, based upon the interceptor-allocation schemes embedded in most engagement models. Although not analyzed in this paper, we suspect that similar conclusions might be drawn about the algorithms used in the engagement models for allocation of ERIS and HEDI interceptors.

Conversely, the results of Chapter III lead us to conclude that most popular engagement models contain embedded allocation schemes which are appropriate for describing the boost and post-boost phases of the battle for the cases in which the architecture is interceptor poor. However, we make no similar statement regarding allocation schemes used in the subsequent phases of the battle.

As described in some of the papers listed in Refs. 3, 4, and 5, however, there is a significant amount of work being devoted to the development of efficient, robust, high-performance algorithms for interceptor-to-target assignment, and many of the approaches appear promising. These are representative, but by no means a complete list of the work being done. Many of the approaches contain the flexibility of the nonlinear approach described in this report. Many of the developers have gone further to examine the midcourse and terminal phase problems, which are of a very different nature from the boost/post-boost phases due to the fact that the "value" of an object is related to many more factors--its probability of being an RV, the value of its predicted aimpoint, the number of

other objects heading toward the same aimpoint, and the remaining time the defense has before it can no longer engage the target.

Finally, we wish to point out that we are not advocating the use of our nonlinear technique of Chapter II in an SDS. This technique was introduced simply to serve as an example of what a nonlinear approach can do for the defense, in terms of enhanced performance. Techniques under development in the SDIO's algorithm development programs will provide similar results and will be far more computationally efficient.

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## APPENDIX: DETAILS OF CALCULATIONS

In this appendix the architecture used in the calculations of Chapters III and IV is described. The offense consists of 900 boosters with 4200 RVs as described in Table 1. All boosters are launched simultaneously on minimum-energy trajectories.

The defense consisted of 360 CVs, arranged in 15 rings with 24 CVs per ring. Adjacent ascending nodes were 24 degrees apart in longitude. The phasing of the CVs in adjacent rings was set to zero, so that comparable CVs in each ring were at the same latitude. The CVs were placed in circular, polar orbits. The number of interceptors per CV and their velocity were varied in the trade studies and thus are given in the figures containing results and the text describing them. In all cases it was assumed that the SBIs had an infinite acceleration, i.e., there was no delay time required for them to reach the burnout velocity. A 100-km earth limb was used, below which no SBI could fly.

A 60-second time delay was assumed for detection, track initiation, and the allocation decision process and then it was assumed that all SBIs were simultaneously committed. Rather than search over the many possible launch times for the enemy-preferred time of launch, the constellation was positioned such that at the moment of launch, a CV in an ascending node was located at 0 degrees latitude and longitude. This served to fix the position of the entire constellation at the time of launch.

**Table 1. Offensive Threat Used in the Calculations Throughout This Report**

Site	Number of Missiles	Launch		Aimpoint		RVs per Missile	Burnout Time, s	Busing Time, s
		Lat.	Lon.	Lat.	Lon.			
1	100	50 N	30 E	40 N	100 W	10	250	200
2	100	50 N	33 E	40 N	100 W	3	220	60
3	100	50 N	36 E	40 N	100 W	1	220	0
4	100	50 N	70 E	40 N	110 W	10	250	200
5	100	50 N	75 E	40 N	115 W	3	220	60
6	100	50 N	80 E	40 N	120 W	1	220	0
7	100	50 N	85 E	40 N	110 W	10	250	200
8	100	50 N	90 E	40 N	115 W	3	220	60
9	100	50 N	95 E	40 N	120 W	1	220	0